ON MODEL-THEORETIC CONNECTED COMPONENTS IN SOME GROUP EXTENSIONS

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This is joint work with Jakub Gismatullin.

For a group G definable in a monster model and for a small set of parameters B, we denote by G_B^{00} the smallest B-type-definable subgroup of bounded index and by G_B^{000} the smallest B-invariant subgroup of bounded index. It was an open problem to find a group G for which $G_B^{00} \neq G_B^{000}$. The first example, found by Conversano and Pillay, is the universal cover $\widetilde{\mathrm{SL}_2(\mathbb{R})}$ of $\mathrm{SL}_2(\mathbb{R})$. Among other properties, their proof uses the fact that $\widetilde{\mathrm{SL}_2(\mathbb{R})}$ is a central extension of $\mathrm{SL}_2(\mathbb{R})$ by $\mathbb{Z}$ given by a definable 2-cocycle $h \colon \mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R}) \to \mathbb{Z}$ with finite image.

This led us to the following general question.

Question. When does an extension $\widetilde{G}$ of a group G by an abelian group A satisfy $\widetilde{G}_B^{00} \neq \widetilde{G}_B^{000}$ for some parameter set B (working in a monster model)?

We consider this problem in a general algebraic context, i.e., without assuming that $\widetilde{G}$ is a universal cover of a topological group or that G is definable in an o-minimal structure. The only restriction that we make is the assumption that the 2-cocycle $h \colon G \times G \to A$ defining our extension is definable and has finite image.

Our goal was to find sufficient (and necessary, at least in some situations) conditions on h for which $\widetilde{G}_B^{00} \neq \widetilde{G}_B^{000}$, and our main theorem provides such conditions.

Using this theorem, we obtain new classes of examples of extensions (including the example of Conversano and Pillay) for which $\widetilde{G}_B^{00} \neq \widetilde{G}_B^{000}$, e.g. some central extensions of $\mathrm{SL}_2(k)$ for k being any ordered field as well as some central extensions of non-abelian free groups, of surface groups, etc. In order to apply our theorem to get these new examples, we use Matsumoto-Moore theory and various quasicharacters considered in bounded cohomology.

During my lecture, I will discuss the main theorem and I will present some of the examples that we obtained. If time permits, I will also mention a result providing the first known example of an extension given by a 2-cocycle with infinite image where the two connected components differ.