▸ ALEX WILKIE, *Complex continuations of functions definable in $\mathbb{R}_{an,exp}$ with a diophantine application.*

Diophantine properties of subsets of $\mathbb{R}^n$ definable in an o-minimal expansion of the ordered field of real numbers have been much studied over the last few years and several applications to purely number theoretic problems have been made. One line of inquiry attempts to characterise the set of definable functions $f : \mathbb{R} \to \mathbb{R}$ having the property that $f(\mathbb{N}) \subseteq \mathbb{N}$. For example, a result of Thomas, Jones and myself shows that if the structure under consideration is $\mathbb{R}_{exp}$ (the real field expanded by the exponential function) and if, for all positive r, $f(x)$ eventually grows more slowly than $exp(x^r)$, then f is necessarily a polynomial with rational coefficients. In this talk I shall improve this result in two directions. Firstly, I take the structure to be $\mathbb{R}_{an,exp}$ (the expansion of $\mathbb{R}_{exp}$ by all real analytic functions defined on compact balls in $\mathbb{R}^n$) and secondly, I allow the growth rate to be $x^N \cdot 2^x$ for arbitrary (fixed) N. The conclusion is that $f(x) = p(x) \cdot 2^x + q(x)$ for sufficiently large x, where p and q are polynomials with rational coefficients.

I should mention that over ninety years ago Pólya established the same result for entire functions $f : \mathbb{C} \to \mathbb{C}$ and that in 2007 Langley weakened this assumption to f being regular in a right half-plane of $\mathbb{C}$. I follow Langley's method, but first we must consider which $\mathbb{R}_{an,exp}$-definable functions actually have complex continuations to a right half-plane and, as it turns out, which of them have a definable such continuation.